ISSN 2229-5518

Scattering of thermo elastic waves at wavy boundary of a micropolar semi-space.

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Abstract -In this work we discusses the scattering of plane thermo-elastic waves at wavy boundary of a micropolar semi-space . Method of small perturbations has been used. The analyses shows that surface wave breaks into three parts. Rayleigh wave with velocity C scattered waves with velocity of propagation $\frac{cw}{w-rc}$ and $\frac{cw}{w+rc}$ .It is also seen that scattered wavevelocity depends on the wave length and also on the wavy nature of the boundary.

INTRODUCTION- The scattering of elastic waves at a rough surface has been discussed by a number of workers [1,25,86] in classical theory of elasticity.Considerations of thermal heating and the resulting thermoelastic field has been a subject of interest for many years.

Literature survey shows that the corresponding analysis for micropolar elastic solid has not been dicussed probably because om much mathematical complexities. In the present analysis we discuss scattering of plane waves in a micropolar elastic half space bounded by sinu-soidal surface under the following assumption ;

(i) Semi-space is homogeneous, free from any heat source

(ii) The surface is slightly rough i.e. the amplitude and curvature of the roughness are sufficiently small. The sinu-soidal model of roughness has been considered. The method of small perturbation is used to investigate the wave propagation.

Mathematical model;

We consider a micropolar elastic half space- $-\infty \prec (x_1, x_3) \prec \infty, x_2 \geq hf(x_1)$ bounded by a surface $x_2 = hf(x_1)$, where $f(x_1) = \frac{\pi}{r\sin(rx_1)}$ and h $\langle\langle 1$ represent a small perturbation parameter such that h^2 and its higher order terms are neglected (i.e the surface is slightly wavy) and we assume that

The wavy boundary has a normal traction of the concentrated type , zero shear and zero couple stress.

$$\sigma_{nn} = p(\delta s), \sigma_{ns} = 0, \mu_{sp} = 0 \quad \text{for} \quad x_2 = hf(x_1) \qquad \ldots\ldots\ldots\ldots\ldots\ldots(1.1)$$

Where σ_{nn} is the normal stress component for the wavy boundary in the direction of normal to the curve , σ_{ns} is the shear stress component for the wavy boundary along the curve , μ_{sp} is the couple stress component in the direction of binormal to the curve.

(i) The temperature and deformation fields do not depend on the variable x_3 .

(ii) The surface under consideration dissipates according to Newton,s law of cooling

$$\frac{\partial T}{\partial n} + HT = 0 \qquad (1.2)$$

Basic Equation

Nowacki (1969) showed that when displacement $\vec{u} = (u_1, u_2, u_3)$ and rotation $\vec{w} = (w_1, w_2, w_3)$ depend on the variables x_1, x_2 and t ,we face two mutually independent systems of equations ;

$$(\mu+\alpha)\nabla^2 u_2 + (\mu+\lambda-\alpha)\partial_1 e_1 + 2\alpha\partial_2\omega_3 = \rho\ddot{u}_1 + \upsilon\partial_1 T$$

$$(\mu+\alpha)\nabla^2 u_2 + (\mu+\lambda-\alpha)\partial_2 e_2 - 2\alpha\partial_1\omega_3 = \rho\ddot{u}_2 + \upsilon\partial_2 T$$

$$(\gamma+\varepsilon)\nabla^2\omega_3 - 4\alpha\omega_3 + 2\alpha(\partial_1 u_2 - \partial_2 u_1) = j\ddot{w}_3 \qquad (1.3)$$

And

$$(\gamma+\varepsilon)\nabla^2\omega_1 + (\gamma+\beta-\varepsilon)\partial_1\chi_1 - 4\alpha\omega_1 + 2\alpha\partial_2 u_3 = j\ddot{w}_1$$

$$(\gamma+\varepsilon)\nabla^2\omega_2 + (\gamma+\beta-\varepsilon)\partial_2\chi_1 - 4\alpha\omega_2 - 2\alpha\partial_1 u_3 = j\ddot{w}_2 \qquad (1.4)$$

$$(\mu+\alpha)\nabla^2 u_3 + 2\alpha(\partial_1\omega_2 - \partial_2\omega_1) = j\ddot{\rho}_3$$

Where $\lambda, \mu, \alpha, \beta, \gamma, \varepsilon$ are the elastic constants of the micropolar material, ρ is the density, J IS The rotational inertia and dots denote the time derivative. The following notation have been used in the equations (1.3) and (1.4)

$$\nabla^2 = \partial^2{}_1 + \partial^2{}_2 \;, e_1 = \partial_1 u_1 + \partial_2 u_2 \quad , \chi_1 = \partial_1\omega_1 + \partial_2\omega_2$$

$\upsilon = (3\lambda + 2\mu)\alpha_t$ and α_t is the coefficient of linear thermal expansion.

T = Temperature distribution in the material satisfying coupled heat equation in the absence of any heat source .

$$(\nabla^2 - \frac{1}{\chi}\partial_t) - \eta \ \text{div}\ \vec{u} = 0 \qquad (1.5)$$

Where $\chi = \dfrac{k}{c_e}, k$ denoting the heat conducting coefficient and C_e the specific heat at constant deformation. $\eta = \dfrac{\theta_0}{k}$, θ_0 being the absolute temperature of the natural state.

The latter system (1.4) is unperturbed by the thermal field, As such we shall consider types of waves governed by the former system (1.3), with boundary condition given by (1.1).

Equation in terms of potential

We introduce the elastic potentials $\phi and \psi$ connected with the displacements u_1 and u_2 by

$$u_1 = \partial_1\phi + \partial_2\psi$$

$$u_2 = \partial_2\phi + \partial_1\psi \qquad (1.6)$$

Inserting (1.6) in (1.3) ,we get the following

$$\left(\nabla^2 - \frac{1}{K_1^2}\partial_t^2\right)\phi = mT \qquad (1.7)$$

$$\left(\nabla^2 - \frac{1}{K_2^2}\partial_t^2\right)\psi + \alpha_2\omega_3 = 0 \qquad (1.8)$$

$$(\nabla^2 - 2\alpha_4 - \frac{1}{k_4^2})\omega_3 - \alpha_4\nabla^2\psi = 0 \qquad (1.9)$$

Where $\left\{k_1^2, k_2^2, k_4^2\right\} =$

$$\left\{k_1^2, k_2^2, k_4^2\right\} = \left\{\frac{\lambda+2\mu}{\rho}, \frac{\mu+\alpha}{\rho}, \frac{\gamma+\varepsilon}{j}\right\}$$

$$\left\{\alpha_2, \alpha_4\right\} = 2\alpha\left\{\frac{1}{\mu+\alpha}, \frac{1}{\gamma+\varepsilon}\right\}$$

$$m = \frac{3\lambda+2\mu}{\lambda+2\mu}\alpha_t$$

Eliminating Tfrom the equations (1.5) and (1.7).
We get the wave equation

$$\left(\nabla^2 - \frac{1}{k_1^2}\partial_t^2\right)\left(\nabla^2 - \frac{1}{\chi}\partial_t\right)\phi - \frac{\varepsilon'}{\chi}\partial_t\nabla^2\phi = 0 \qquad (1.10)$$

Where $\varepsilon' = \eta m \chi$

Also from the coupled equations (!.3) and (1.9) we get two other wave equations.

$$\left[\left(\nabla^2 - \frac{1}{k_2^2}\partial_t^2\right)\left(\nabla^2 - 2\alpha_4 - \frac{1}{k_4^2}\partial_t^2\right) + \alpha_2\alpha_4\nabla^2\right]\psi = 0$$

(1.11)

$$\left[\left(\nabla^2 - \frac{1}{k_2^2}\partial_t^2\right)\left(\nabla^2 - 2\alpha_4 - \frac{1}{k_4^2}\partial_t^2\right) + \alpha_2\alpha_4\nabla^2\right]\omega_3 = 0$$

(1.12)

It may be noticed that equation (1.10) represent longitutudinal wave where as equations (1.11) and (1.12) represent transverse and tortional waves respectively.

Conclusion ; After solving the above problem, the fourier transformation of components of displacements and microrotation are given by

$$\overline{u_1} = \overline{u_1^0} + \overline{u_1'}$$

$$\overline{u_2} = \overline{u_2^0} + \overline{u_2'}$$

$$\overline{\omega_3} = \overline{\omega_3^0} + \overline{\omega_3'}$$

Where ($\overline{u_1^0}$, $\overline{u_2^0}$, $\overline{\omega_3}$) represent transformed displacement vector for plane boundary $x_2 = 0$ and ($\overrightarrow{u_1'}$, $\overline{u_2'}$, $\overline{\omega_3'}$) arises due to the wavy boundary. It is seen that surface waves breaks into three parts

(a) Rayleigh wave with velocity c

(b) scattered waves with velocity of propagation $\dfrac{cw}{w-rc}$ and $\dfrac{cw}{w+rc}$

(c) The scattered waves depend on the wave length as well as on the wavy nature of the boundary.

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